

Natural Deduction Guide

Rules of Inference

(MP) $(p \rightarrow q), p / q$	(MT) $(p \rightarrow q), \sim q / \sim p$
(HS) $(p \rightarrow q), (q \rightarrow r) / (p \rightarrow r)$	(DS) $(p \vee q), \sim p / q$
(Add) $p / (p \vee q)$	(Con) $p, q / (p \bullet q)$
(Simp) $(p \bullet q) / p$	(CD) $(p \vee q), (p \rightarrow r), (q \rightarrow s) / (r \vee s)$

Rules of Replacement

(DN) $\sim \sim p :: p$	(Imp) $(\sim p \vee q) :: (p \rightarrow q)$
(Cont) $(p \rightarrow q) :: (\sim q \rightarrow \sim p),$	(DM) $\sim(p \bullet q) :: (\sim p \vee \sim q)$ $\sim(p \vee q) :: (\sim p \bullet \sim q)$
(Exp) $(p \rightarrow (q \rightarrow r)) :: ((p \bullet q) \rightarrow r),$	(Equiv) $(p \leftrightarrow q) :: ((p \rightarrow q) \bullet (q \rightarrow p))$
(Taut) $p :: (p \bullet p)$ $p :: (p \vee p)$	(Comm) $(p \vee q) :: (q \vee p)$ $(p \bullet q) :: (q \bullet p)$
(Asc) $(p \vee (q \vee r)) :: ((p \vee q) \vee r)$ $(p \bullet (q \bullet r)) :: ((p \bullet q) \bullet r)$	(Distr) $(p \bullet (q \vee r)) :: (p \bullet q) \vee (p \bullet r)$ $(p \vee (q \bullet r)) :: (p \vee q) \bullet (p \vee r)$

Conditional Proof (CP): Assume the antecedent p of $(p \rightarrow q)$. Derive q using only premises and valid rules of inference, then enter $(p \rightarrow q)$ on the line after q , with the justification (CP) and the lines used.

Indirect Proof (IP): Assume the negation of the conclusion. So, if the conclusion is p , assume $\sim p$. Use $\sim p$, the premises, and any valid rules of inference to derive a contradiction \perp of the form $(r \bullet \sim r)$. On the line after the contradiction is formed enter p with justification (IP).